# The Physics of Xylophones

The Science Behind Bar Tuning, Bar Stringing, and Resonator Tubes

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## Abstract

The sound of the xylophone is governed by the natural frequencies of the wooden bars when they are struck but is also largely affected by where the bars are strung and the length of the resonator tubes below. This paper will discuss the physics behind these three components and how they ultimately affect the overall sound of the instrument. The methods discussed in this paper result in a relatively well-tuned, but still imperfect, instrument; it may be possible to create a better sounding instrument by exploring different bar shapes in further experiments.

## Introduction

Xylophones are much more complex instruments (with relation to both physics and actual construction) than they may seem upon first glance. In order to build a concert-grade xylophone, one must have a very solid understanding of the physics behind the instrument. Xylophone construction requires a deep understanding of the physics of sound, waves and materials, and the ability to utilize experimental methods in a multitude of creative ways.

An understanding of the physics behind xylophones necessitates a strong base of knowledge with regard to the basic principles of waves and sound. To start, we know that, for all waves, the relationship between wave speed, frequency, and wavelength is  $v = \lambda f$ . By convention, we define that lower notes/pitches have lower frequencies, and higher notes have higher frequencies. Sound waves travel as longitudinal waves, also known as pressure waves; when a sound wave travels through the air, it displaces air particles, resulting in higher pressured sections of air and lower pressured sections of air relative to the axis of travel ("Sound is a Pressure Wave," 2006). As these variations in pressure travel through the air, they enter the ear and strike the membrane of the eardrum, making it vibrate. These vibrations are then transferred to the cochlea, where the mechanical signals are turned into electrical signals within the neurons; these signals are then transferred to the brain by way of the auditory nerve, and the brain interprets them as sound (Errede). Higher frequencies have more frequent compressions than their lower counterparts, and therefore hit the ear with high pressure more often; we know this because all sound travels through the air at the same speed, so in order for the equation  $v = \lambda f$  to hold true, higher frequencies must have shorter wavelengths and vise versa.

Sound waves result from vibrating objects. Nearly all objects begin to vibrate when they are hit, strummed, plucked, or excited in some manner. When mechanical energy is put into an object, the object oscillates between states of potential energy and kinetic energy until eventually this vibration comes to a stop due to energy loss in the system (typically caused by friction). Every object, depending upon the properties of its material and its shape, has a dominant natural frequency; in the language of acoustics, this is called the *fundamental* frequency. There are many factors that determine an object's fundamental frequency, and each of these factors must either influence an emitted wave's velocity or wavelength, because  $f = v/\lambda$  ("Natural Frequency," 2006). Examples of these factors include the material's shape, density and stiffness (Young's modulus). Some objects, when excited, emit a single frequency, such as a tuning fork. Most objects, however, have a *set* of natural frequencies that they emit when struck. Frequencies beyond the fundamental (which is the lowest and most dominant tone) are called overtones. Oftentimes objects vibrate at a set of frequencies that have no mathematical relationship; musically, this tends to result in the emission of dissonant, discordant sounds. Musical instruments, however, are built and tuned so that they vibrate with a set of frequencies that are typically related by integral factors; this results in a rich, choral sound ("Natural Frequency," 2006).

## The Physics of Tuning the Bars

Before anything else in xylophone design, the bar material must be selected. As discussed previously, an object's material properties drastically influence the natural frequencies at which it vibrates, so proper material selection is imperative. There are several materials commonly used in the fabrication of xylophone bars - Kelon, Padauk, and Honduras Rosewood (Bremaud, et al). As noted above, density and stiffness are the dominant properties that determine the selection of a material for the bars, but there are other properties that also affect the tonal quality (e.g., the decay constant). These three most common xylophone bar materials have similar densities and modulus values, however, Honduras Rosewood stands out as the superior material because of its unique secondary properties that render it ideal for concert grade instruments.

The major challenge in xylophone construction is the task of tuning the bars. When one sets out to build a xylophone, the intended frequency modes for each bar are relatively simple to calculate. We know that an octave spans from any given frequency to two times that frequency, and that there are 12 different notes between each octave in an equally tempered scale. The ratio between any given pair of these notes is  $2^{1/12}$  (Allen). In the United States, it is accepted that the ratio between the fundamental frequency and the first two overtones (which is practically all that is achievable during tuning) is 1:3 and 1:6; so if the fundamental frequency is 440 Hz, then the first overtone is 1320 Hz and the second overtone is 2640 Hz (Youhass). Given these few constraints, one can easily calculate the desired fundamental and overtones for each bar.

The problem, however, rests in the fact that there is no simple way to derive bar shape from the desired frequencies. Each bar's intended fundamental frequency and overtones create a chord that is most often achieved with a single or double scalloped undercut. Theoretically, there are infinitely many possible bar shapes that would emit the desired frequencies, so it is necessary to mathematically constrain the problem of determining the exact bar shape in order to make it tractable. Second and third order polynomials are often chosen to describe the undercut shape, due to their relative mathematical simplicity. For simple shapes, like tubes and strings, the fundamental frequency and overtones for a given shape and material can be predicted by a series of relatively simple mathematical equations (Giancoli). In mechanical engineering, there is a concept known as *Beam Theory*, which allows for the prediction of vibrational modes in long, thin objects with uniform cross sections (also known as *prismatic* beams) (Haque). Beam theory could be used quite simply to determine the exact shape of xylophone bars if they had uniform cross sections, but because of their scalloped undercuts, their cross sections are nonuniform.



Prismatic beams have uniform cross sections. Credit: Rich Wickstrom

It takes the keen sense of an engineer or physicist to realize that beam theory can still be used to predict vibrational modes of nonuniform objects like a xylophone bar; a non-uniform object can be approximated as a series of end-to-end prismatic beams of varying thicknesses. Essentially, a xylophone bar can be visualized as many tiny rectangular bars pieced together into one scalloped shape. Beam theory can then be used to calculate the vibrational modes of each segment, and a mathematical technique called *receptance substructuring* can then be used to model the connections between them (Stone).

Another problem arises, however, when one overcomes the first hurdle. The technique described above can be used to determine vibrational modes given a certain bar shape, but cannot determine bar shape given desired vibrational modes. This results in a challenging problem, considering the initial task was to determine the ideal bar shapes to produce a predetermined range of frequencies. The best way to overcome this problem is to essentially work it in reverse. The idea is to start with a reasonable guess for a bar shape, compute the resulting vibrational modes, and then modify the bar shape based on a difference between the computed vibrational modes and the desired vibrational modes. This technique is generally referred to as *constrained non-linear minimization*. A computer algorithm is used to iteratively minimize the error between the desired frequencies and the computed frequencies; for all practical purposes, this is an automated guess and check technique. When the algorithm is complete, it determines the necessary undercut to yield the desired frequencies, and generates an image of the ideal bar.



A computed bar shape from the algorithm program. Credit: Rich Wickstrom

# The Physics of Stringing the Bars

After each of the 44 bars is tuned, a method to support them without dampening the vibrational modes must be determined. The most common approach is to drill holes across the width of the bar so that a taught string can be used to support the bar. Care must be taken to determine the hole locations, because it is necessary to minimize the damping effect of the strings on the vibrational modes. We know that standing waves have both *antinodes*, where the displacement is at a maximum, and *nodes*, where the displacement is zero. The optimal place to string the bars would logically be the nodes,

because holes placed at the unwavering nodes would minimize the effects on the bar's vibration and frequencies. However, it is important to quantify the actual effect of drilling the holes at the nodal locations. In order to evaluate the effect of drilling the holes, it is first necessary to determine the location of the nodes for each bar; this can be done via a simple experiment.

# **Determination of the Nodal Locations**

Purpose: To locate each bar's nodes Materials: Wood, Rubber bands, Salt, Mallet, Xylophone bars Procedure:

- Create a "suspension box" on which to test the bars by building a three-sided, trough-shaped, wooden box and placing rubber bands around the box to suspend the bar.
- 2. Set the first bar on the rubber bands at the anticipated nodal locations.
- 3. Sprinkle salt on the suspended bar.
- 4. Repetitively strike the bar with a mallet until the salt migrates to form tight lines over the bar's nodal points.
- 5. Mark each nodal line with a pencil.
- 6. Repeat steps 1-5 with all of the remaining bars.

Creating a wooden "suspension box" with rubber bands helps to minimize the damping of the bars' frequencies when running this experiment. When the bar is struck with a mallet, the salt migrates to the bars' nodal points because the bar vibrates minimally in those spots. For well-shaped bars, the salt forms tight, clean lines at either end of the bar.



A bar with salt before and after being struck.Credit: Rich Wickstrom

Before running this experiment, one might have thought that the nodal lines would be perpendicular to the long axis of the bar. This, however, rarely happens, because of the grain of the wood. In mechanical engineering, this is referred to as "the anisotropic nature of the material" (Mingming).

After determining the location of the nodes, the next step is experimenting to find out just how much drilling holes will affect the frequency modes of the bars. This too can be achieved through a relatively simple experiment. First, a practice bar without holes is placed on the suspension box. A microphone is then hooked up next to the bar. Next, a mallet is used to strike the bar, and the subsequent recording is analyzed by software that spectrally analyzes the bar to determine its emitted frequencies.

Selected Run	76
Bar:	01 - F4 - 350.8
Fund Hz (cents):	355.4 (+22)
2nd Hz:rat (cents):	1046.4 : 2.983 (-10)
3rd Hz:rat (cents):	2512.0 : 7.161 (+306)

Spectral analysis of a practice bar prior to holes being drilled. The fundamental frequency and the first two overtones can be seen. Credit: Rich Wickstrom

After holes are drilled, the bar is then placed back on the rubber bands, struck, recorded, and analyzed again.

	Stats
Selected Run:	77
Bar:	01 - F4 - 350.8
Fund Hz (cents):	355.4 (+22)
2nd Hz:rat (cents):	1044.6 : 2.978 (-13)
3rd Hz:rat (cents):	2511.7 : 7.160 (+306)

Spectral analysis of a practice bar with holes. Credit: Rich Wickstrom

This data reveals that holes drilled at a bar's nodes have relatively negligible effects on the bar's vibrational modes. The most important frequency of the three

analyzed, the fundamental, was not changed at all; this greatly supports the expectation that the drilling the holes at the nodes would have minimal effect. The frequency most affected by this change was the second partial (the first overtone), which went a bit flat. This change in frequency, however, is hardly noticeable to the untrained ear and can be fixed with some minor tuning after the holes are drilled. The combination of the salt experiment with the spectral analysis confirms that the holes for each bar should be drilled at the nodes.

## The Physics of the Resonator Tubes

So we now know how to tune the bars and where to string them. What's next? The second most important component of xylophones (behind the bars) is the resonator tubes. Resonator tubes are traditionally hollow metal cylinders with end stoppers and are mounted below the bars. Essentially, resonator tubes serve as amplifiers for certain frequencies. Stopped tubes only resonate with frequencies that have wavelengths that are four times the length of the tube and odd multiples beyond that (Giancoli). Resonator tubes amplify only the frequencies that they resonate with; they boost these special frequencies by trapping the normally wasted downward-directed sound energy and essentially "bouncing" it back upwards in a flat plane of sound.



Stopped tubes only resonate with certain frequencies. Credit: Giancoli

When a xylophone bar is hit with a mallet it reacts by vibrating- we know this already. A bar, however, does not just vibrate in the up and down direction (the transverse mode) as one might expect. They also compress and expand along the long axis (longitudinal mode) and form a twisting motion (torsional mode) (LaFavre). Xylophone builders do not really have the capability to tune any mode other than the transverse mode, so the sound of their instruments is therefore tainted by unwanted frequencies caused by the longitudinal and torsional modes. The use of resonator tubes, however, essentially mitigates this problem. Frequencies emitted by the longitudinal and torsional modes are rarely boosted by resonator tubes, because it is uncommon for them to ever be odd harmonics of the fundamental. Because of this, these unwanted frequencies can barely be heard in the sound of the final instrument. Recall that xylophones are tuned to a 1:3:6 frequency relationship, where the first overtone is three times the fundamental and the second overtone is six times the fundamental. Because resonator tubes are made to only resonate with the fundamental and odd multiples of it, only the fundamental frequency and first overtone are amplified. When presented with this information, it is important to realize that resonator tubes do not only function as amplifiers, but they also function as a means to shape the ultimate sound of the instrument.

## Conclusion

A xylophone is a very complex instrument that, when played, exemplifies many principles of physics. To summarize, when a mallet strikes a bar, mechanical energy from the mallet is transferred into the bar, causing it to vibrate at its natural frequencies; these frequencies are determined by material properties and bar shape. As waves from the bar travel into the resonator tube below, certain frequencies are amplified, and others are not; this is determined by the length of the resonator tube. Waves from both the resonator tube and the bar then travel through the air as variations in air pressure and eventually reach the ear, where they are perceived as vibrations and eventually interpreted as sound.

In addition to discussing bar stringing and resonator tubes, this paper describes the process of determining bar shape given desired frequencies. In order to solve this problem, the broad mathematical situation must be constrained. The technique described in this paper does so by limiting the potential undercuts to only those that fit the cubic model. This accomplishes the initial task, but does so imperfectly; the second overtone (third partial) is left largely untuned due to the limitations in this undercut shape.



The third partial is left largely untuned due to the limitations of a cubic undercut. Credit: Rich Wickstrom

Exploring undercuts that mirror higher order functions may yield better control over the desired sound properties in the resulting bar. In the future, experiments should be conducted to determine if higher order polynomial undercuts could gain better control over the higher overtones and sound qualities like amplitude and sustain.

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## Summaries of Each Source

1. "Equal Temperaments as Mathematical Series" :

In an equally tempered scale, math can be used to describe the relationship between the 12 notes in an octave. The ratio between any given two sequential notes is  $2^{1/12}$ .

- "Vibrational Properties of Tropical Woods with Historical Uses in Musical Instruments.": As compared to other woods, Hondouras Rosewood has an extremely high density paired with a very low decay factor (meaning it "rings" for a very long time when hit). This makes it the best wood for concert grade instrument building.
- 3. "The Human Ear Hearing, Sound Intensity and Loudness Levels" : Humans hear by sensing the vibrations of a pressure wave and turning that mechanical signal into interpretable electrical signals.
- "Sound." Physics Principles With Applications : Equations can be used to determine what frequencies strings and tubes will emit/ resonate at.
- "The Timoshenko Beam Theory and Its Extension" : A series of complicated equations known as beam theory can used to predict the vibrational modes of a prismatic beam.
- 6. "Tuning the Marimba Bar and Resonator" : A summary of the physics and building techniques behind marimba bars.
- 7. "The Physics of Music and Musical Instruments" : An explanation of the physics behind a variety of musical instruments.
- 8. Automatic Multi-Modal Tuning of Idiophone Bars: An algorithm can be used to predict a xylophone bar's emitted frequencies given its shape.
- "Natural Frequency": The frequency at which something naturally resonates depends upon the object's material properties and shape.
- 10. "Sound Is a Pressure Wave": Sound travels through the air as a pressure (longitudinal) waves.
- 11. The Receptances of Beams, in Closed Form, including the Effects of Shear and Rotary Inertia:

Beam theory can be used to estimate the emitted frequencies of non-prismatic objects by approximating them as many prismatic beams put together and

summarizing the relationship between them with complex mathematical equations.

12. "The Art and Science of Mallet Instrument Tuning": Description on the techniques behind idiophone tuning.